



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

(See Vol. V, Nos. 6-7). Now $h=(c/R)(R-r)$.

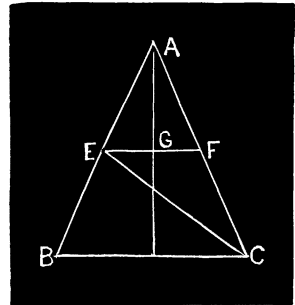
$\therefore$ Surface of which EBU is a projection $= \frac{\pi}{R} \sqrt{(R^2 + c^2)} [R^2 - \frac{1}{2} \sqrt{(Rr)(R+r)}]$.

S = surface of which $EAFU$ is the projection.

$$\begin{aligned} \therefore S &= \pi R \sqrt{(R^2 + c^2)} - \frac{\pi}{R} \sqrt{(R^2 + c^2)} [R^2 - \frac{1}{2} \sqrt{(Rr)(R+r)}] \\ &= \frac{\pi \sqrt{(Rr)(R+r)} \sqrt{(R^2 + c^2)}}{2r}. \end{aligned}$$

If the two remaining points are taken anywhere on the surface S the section will be a complete ellipse.

$$\begin{aligned} \therefore \text{The chance} &= \frac{\int_0^R \pi^2 (R^2 + c^2) r (R+r)^2 dr}{4\pi^2 R^3 (R^2 + c^2) \int_0^R dr} \\ &= \frac{1}{4R^4} \int_0^R (R^2 r + 2Rr^2 + r^3) dr = \frac{17}{48}. \end{aligned}$$



150. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

If the length of a circular arc be b and the radius vary uniformly, what is the average area of all the segments possible?

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The least area is $=0$ when R = radius is ∞ . The greatest area is the area of a circle, $R=b/(2\pi)$, circumference $=b$, $\therefore$ area $=b^2/(4\pi)$.

$$\therefore \text{Average area} = \frac{1}{2} \left(\frac{b^2}{4\pi} + 0 \right) = \frac{b^2}{8\pi}.$$

The problem may also be solved as follows: Area of segment

$$= \frac{1}{2} Rb - \frac{1}{2} R^2 \sin(b/R). \quad \text{Average area} = \Delta = \frac{1}{2} \int_{b/2\pi}^{\infty} (Rb - R^2 \sin b/R) dR / \int_{b/2\pi}^{\infty} dR.$$

Let $b/R=y$, then limits of y are 0 and 2π .

$$\begin{aligned} \therefore \Delta &= \frac{1}{2} b^2 \int_0^{2\pi} \left(1 - \frac{1}{y} \sin y \right) \frac{dy}{y^3} / \int_0^{2\pi} \frac{dy}{y^2} \\ &= \frac{1}{2} b^2 \left\{ \frac{\left[-\frac{1}{2y^2} + \frac{\sin y}{3y^2} + \frac{\cos y}{6y^2} - \frac{\sin y}{6y} + \frac{1}{6} \left(\log y - \frac{y^2}{2 \cdot 2!} + \frac{y^4}{4 \cdot 4!} - \frac{y^6}{6 \cdot 6!} + \dots \right) \right]_0^{2\pi}}{\left[-\frac{1}{y} \right]_0^{2\pi}} \right\} \\ &= b^2 / 8\pi. \end{aligned}$$